# Lecture 8

## The Law of Total Probability and Bayes Rule

Let’s look at a simple case of what happens when we are talking about a couple having 2 children.

Visually, what are the options (**Probability Tree**)?

Now, let’s look at formulaically what is happening:

The fundamental principle used here is the multiplication principle:

in conjunction with the Law of Total Probability:

to finally apply Bayes Rule (in the case of 2 events):

So, what we'd be given is:

$$P(A|B) \quad [or\ P(A^c|B)\ which\ equals\ 1 - P(A|B)\ ]$$
$$P(A|B^c) \quad [or\ P(A^c|B^c)\ which\ equals\ 1 - P(A|B^c)\ ]$$
$$P(B) \quad [or\ P(B^c)\ which\ equals\ 1 - P(B)]$$

**Don't get caught up in the variables A,B!**

If it helps, we can think of these verbally instead:

We’d be asked to find:

$$P(A)$$
$$P(B|A)$$

Example 1 – Coffee

*CNBC just posted an article last night about how Dunkin is “crushing” Starbucks this year in the stock market. The probability of Dunkin Donuts stock price increasing is 50%. The probability of Starbucks stock price increasing knowing that Dunkin Donuts’ did is 40%. The probability of Starbucks stock price increasing knowing that Dunkin Donuts’ did not is 1%.*

a. Write down the 3 pieces of given information in terms of the probability of events A (Starbucks stock price increasing) and B (Dunkin Donuts stock price increasing).

b. Draw a probability tree for this problem.

c. What is the probability that Starbucks’ stock price increases?

d. Given that Starbucks' stock price increased, what is the probability that Dunkin Donuts' stock price increased?

Example 2 – Wine Pairing

*In pairing which wine to drink with which meal, the general rule of thumb is to pair the color of the wine to the color of the main course. So, knowing that the person ordered fish the probability of the person pairing their meal with a white wine (properly) is 78%. Given that the person did not order fish, the probability of the person pairing their meal with a white wine is .34. The probability of the person ordering fish is .27.*

a. Write down the 3 pieces of given information in terms of the probability of events F (the person ordered fish) and W (the person paired their meal with white wine).

b. Create a probability tree for this problem.

c. Find the probability that the person paired their meal with white wine.

d. Find the probability that given they ordered white wine, they properly paired their meal.

Example 3 – Polygraph Tests

*Polygraph tests (lie detector tests) are often routinely administered to employees or prospective employees in sensitive positions (or every time on Criminal Minds…). According to studies of polygraph reliability (Gastwirth, J., 1987. The Statistical precision of medical screening procedures, Statistical Science, 3, 213-222) if a person is lying, the probability that this is detected by the polygraph is .88, whereas if the person is telling the truth, the polygraph is indicating that he/she is telling the truth with probability .86. Now, suppose that on a particular question the vast majority of subjects have no reason to lie, so that 99% tell the truth.*

a. Write down the 3 pieces of given information in terms of the probability of events A (the polygraph reading is positive – meaning the machine thinks the person is lying) and B (the subject is telling the truth).

b. Draw a probability tree for this problem.

c. What is the probability that the polygraph produces a positive reading?

d. Given that a subject produces a positive reading on the polygraph, what is the probability that the polygraph is incorrect and that he/she is telling the truth?

## Random Variables & Probability Distributions

Background: In algebra, when we discuss the unknown we use a variable, X for example. In statistics, we discuss something called a **random variable**.

*Random Variable (RV)* →

Example 1 – Multiple Die Rolls

*If we roll a fair die two times, we have a few random variables we could identify.*

i.

ii.

When it comes to random variables, we distinguish between **Discrete RV's** and **Continuous RV's**.

Very similar to when we discussed quantitative variables in a dataset, the key difference is the **possible values**.

*Discrete Random Variable* →

Properties of Discrete RV's:

i.

ii.

Examples:
-

-

-

-

*Continuous Random Variable* →

i.

## Discrete Random Variables

To summarize the "random behavior" of a discrete RV we use its **probability distribution function** which assigns a probability to each possible value either through calculation using the probability rules we've learned, or through already known info (given info/probability histogram/formula).

Generally:

**Through Calculation (IDENTIFY YOUR SAMPLE SPACE FIRST)**

Example 1 Cont. – 2 Die Rolls

*Find the probability distribution for the random variable X = # 4's rolled.*

Example 2 – Coin Tosses

*A fair coin is tossed 2 times. Let the random variable H represent the number of Heads you get. Find the probability distribution of H.*

Example 3 – Coin Tosses

*A fair coin is tossed until you get one Head and one Tail, or until the coin has been tossed three times. Let the random variable X represent the number of Tails you got. Find the probability distribution of X.*

Example 3 Cont. – Coin Tosses Continued

*Let the RV Y be the number of tosses. Find the probability distribution of Y.*

**Already Known Info - Given Info/Probability Histogram/Formula**

A. *Given Info* → Many times the probability distribution function of X is given to us and is based on long run observations rather than the probability rules that we used for calculation above.

Example 4 – Number of Significant Others

*Choose an adult age 26-30* at random, and let the random variable X be the number of significant others the individual has had in his/her lifetime so far.

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| P(x) | 0.135 | 0.271 | 0.271 | 0.180 | 0.09 | 0.036 | 0.012 | 0.003 | 0.002 |

a. What is the probability that an adult age 26-30 has had at most one significant other?

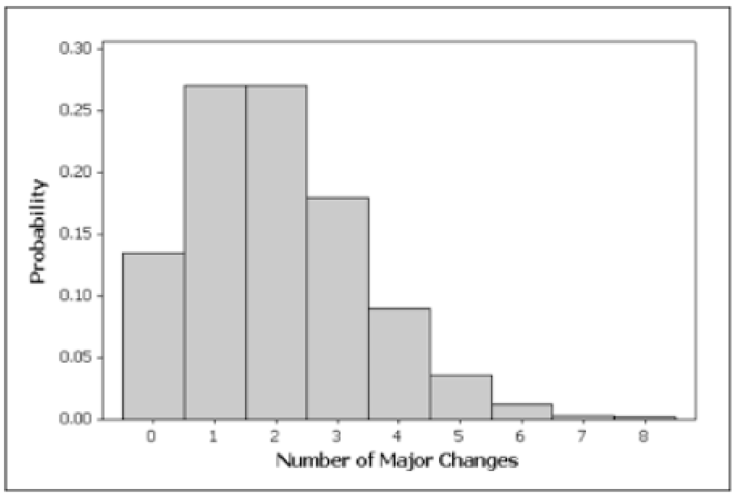
b. Austin's parents are concerned that he has recently broken up with his second girlfriend. Austin claims that this is not unusual. What is the probability that a randomly selected adult age 26-30 has as many or more significant others than Austin?

c. How many significant others would Austin have to have to be considered "unusual" (an event that occurs only 5% of the time or less is pretty unusual)?

d. Given that by age 28, Austin has had two significant others already, what is the probability that he will have no more than four significant others in his life?

*B. Probability Histogram* → the probability distribution function of a discrete RV can also be presented in a probability histogram




We have the following 5 properties about such a graph:

i.

ii.

iii.

iv.

v.

Example – Finding Probabilities Using a Probability Histogram

a. *From the probability histogram above, what is the probability that a randomly selected student makes 1 major change?*

b. *From the probability histogram above, what is the probability that a randomly selected student makes at least 2 major changes?*

*C.* Formula → We can use a mathematical formula as an elegant way to provide the probability distribution for a discrete variable.

Example 1 – Formula Notation

*Consider the given probability distribution for the random variable X measuring the number of hits in a baseball game:*

$$P(x) = \frac{x+2}{20} \quad x = 0,1,2,3,4$$

a. What is the probability that a randomly selected player gets 3 hits?

b. What is the probability that a randomly selected player gets at least 1 hit?

c. Translate the probability distribution from formula form to table form.

**Note: The probability distribution is BOTH THE FORMULA AND THE POSSIBLE X VALUES**

If you leave out the possible values in the formula representation, it's no longer a probability distribution!

Example – Importance of Possible Values

*Let's consider the same formula from above, but this time change the possible values:*

$$P(x) = \frac{x+2}{20} \quad x = 2,4,6,8$$

*What do we notice here?*

**Formal Properties of any Probability Distribution (A Practical Interpretation Kolmogorov's Axioms):**

*i.*

*ii.*

*iii.*

**Note: If any of these are violated, then the formula and possible values together do not represent a probability distribution.**