Lecture 11

Inference

Introduction

Recall the statistics lifecycle:

Some terminology:

Parameter →

Statistic →

Note: Parameters typically have statistic counterparts

We briefly discussed this earlier in the course, but formally there are 3 parameters with corresponding statistics used in the remainder of this course:

	Parameter	**Statistic**
Proportion		
Mean		
Standard Deviation		

Example – Identifying Parameters vs Statistics
Identify the parameter and corresponding statistic in each of the following scenarios:

a) 43% of the U.S. say they like Taylor Swift's music enough to keep her on the radio for longer than 1 minute. To test this, Austin took a cluster sample of 350 UConn statistics students. Of the 350 people in the sample, 159 say they like Taylor Swift's music enough to keep her on the radio for longer than 1 minute.

b) Of those adults aged 26-35, the mean length of time that one owns the same pair of sneakers for athletic activity is 283 days, give or take 11 days. A simple random sample of 134 adults aged 26-35 was followed, and it was found that their mean sneaker length was 291 days, with a standard deviation of 18 days.

A Few Notes on Parameters & Statistics:

i.

ii. ***Sampling Variability*** →

iii.

We must understand the sampling distribution of the statistic in order to infer about the population parameter using the statistic.

Sampling Distribution of the Sample Proportion $\hat{p}$

Here, we will examine the first statistic of interest, the sample proportion $\hat{p}$, and look at its behavior.

Let's look at the situation visually:

In order to discuss this new dataset that we have constructed, the **Sampling Distribution of $\hat{p}$**, we analyze it in the same way we analyze any other dataset: shape, center, and spread.

Statistical theory gives us the following results about the sampling distribution of $\hat{p}$:

Shape:

Since the distribution is Bell-shaped, we then discuss the mean and the standard deviation with regard to measuring central tendency and variability (spread), respectively.

Center:

Spread:

Example – College Exercising

The estimated percent of college students that exercise on a regular basis is 60% some time ago. A health educator suspects that this proportion has increased since then. To check his claim, the educator chooses a random sample of 100 college students and finds that 67 of them exercise on a regular basis.

a) Write down the given information

b) Describe the shape, center, and spread of the sampling distribution of $\hat{p}$. Draw this distribution.

c) Is $\hat{p} = .67$ a reasonable value to get when $p = .60$ (i.e. could it have occurred just by chance due to sampling variability), or is $\hat{p} = .67$ ususally high if $p = .60$ (suggesting that p has actually increased)?

So, the data ($\hat{p} = .67$) do not provide enough evidence to conclude that p has increased.

Example – College Exercising (revised)

The estimated percent of college students that exercise on a regular basis is 60% some time ago. A health educator suspects that this proportion has increased since then. To check his claim, the educator chooses a random sample of 400 college students and finds that 268 of them exercise on a regular basis. Do the data provide evidence of a "real" change?

a) Write down the given information

b) Describe the shape, center, and spread of the sampling distribution of $\hat{p}$. Draw this distribution.

c) With an increased sample size, is $\hat{p} = .67$ a reasonable value to get when $p = .60$ (i.e. could it have occurred just by chance due to sampling variability), or is $\hat{p} = .67$ ususally high if $p = .60$ (suggesting that p has actually increased)?

So, the data ($\hat{p} = .67$) do provide evidence to suggest that p has increased and is thus higher than .60.

d) From here, we can formally calculate the probability of obtaining, just by chance, (i.e. just due to sampling variability) such an unusual (high) $\hat{p}$ or even more unusual (higher):

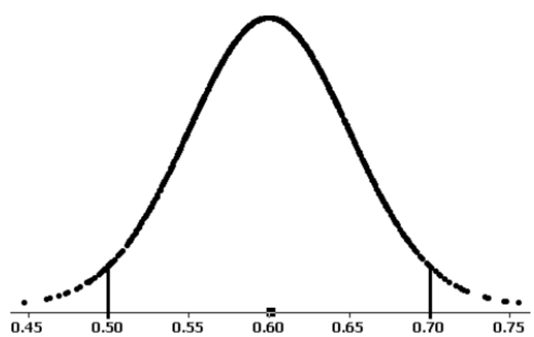
There is less than a 1% chance of getting $\hat{p}$ as high as .67 or higher when $p = .60$ (i.e. it would be extremely unlikely). This is strong evidence that p has probably increased (since if it stayed .60, we would not have observed such a high $\hat{p}$ as we did).

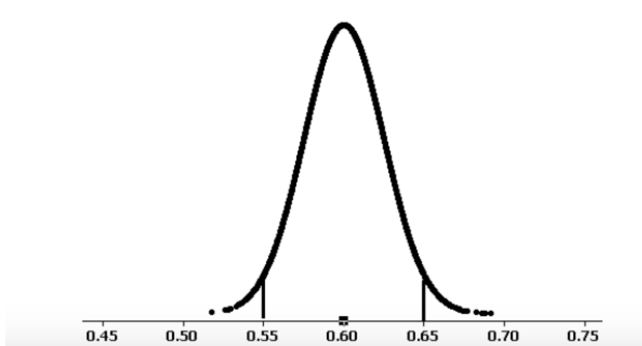
Note: If the sample statistic falls more than 2 standard deviations ABOVE the mean, then this implies the true parameter value is higher than originally claimed. Likewise, if the sample statistic falls more than 2 standard deviations BELOW the mean, then this implies the true parameter value is lower than originally claimed.

What do we notice?

How can we quantify this decrease?

Visually:





Now let's move on to the distribution of another key statistic, the sample mean $\bar{x}$.

The Sampling Distribution of the Sample Mean $\bar{x}$

Visually, what is the big picture?

Again, we aim to describe this distribution using the mean and standard deviation to measure central tendency and variation, respectively.

Statistical theory tell us the following about the sampling distribution of the sample mean $\bar{x}$:

Center:

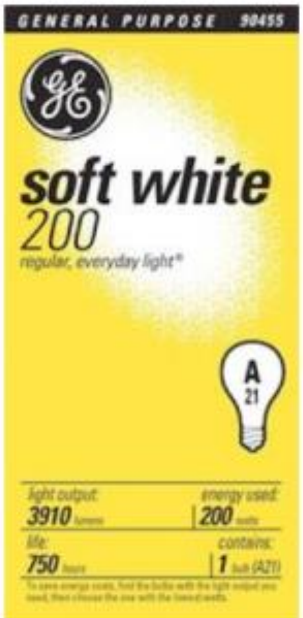
Spread:

Note: In words, this means that the variation from sample mean to sample mean is less than the variation from population individual to population individual by a factor of $\frac{1}{\sqrt{sample\ size}}$.

Shape: **Central Limit Theorem (CLT) →**

Example – Light Bulb Duration

The number of hours that a typical incandescent light bulb lasts before burning out varies from bulb to bulb. One particular type of General Electric Soft White light bulb is advertised to have mean lifetime of 750 hours.



Consumer advocacy groups occasionally test such claims for proof-in-advertising. Suppose some researchers from Consumer Reports wished to test the advertised lifetime of the GE Soft White light bulbs. So, suppose that they selected a reasonably-random sample of $n = 36$ light bulbs (for instance by choosing a random sample of stores in the region, and then, at each store, randomly selecting one of the packages from the shelves). They took the light bulbs to the Consumer Reports laboratory, and the light bulbs were left on until each one burned out. Then, the researchers calculated the average lifetime in the sample to be 744 hours. Suppose the standard deviation of lifetimes of all light bulbs is known to be 12 hours.

a) Write down the given information

b) Describe the shape, center, and spread of the sampling distribution of $\bar{x}$. Draw this distribution. (hint: does the Central Limit Theorem apply here?)

c) Is this sufficient evidence of false advertising? Would Consumer Reports win an expensive lawsuit against GE, embarrassing the company and forcing them to re-tool the plant? Or was Consumer Reports' test result nothing more than natural variation due to sampling, and not a sign of false advertising overall?

So, the data ($\bar{x} = 744$) provide evidence to suggest that μ has decreased and is thus lower than the advertised 750 hours.

d) From here, we can formally calculate the probability of obtaining, just by chance, (i.e. just due to sampling variability) such an unusual (low) $\bar{x}$ or even more unusual (lower):

There is less than a 1% chance (.0013) of getting $\bar{x}$ as low as 744 or lower when $\mu = 750$ (i.e. it would be extremely unlikely). This is strong evidence that μ is probably lower than the advertised 750 (since if it was 750, we would not have observed such a low $\bar{x}$ as we did).

Comments Regarding the Central Limit Theorem

i. The Central Limit Theorem (CLT) applies regardless of the shape of the population distribution from which we sample.

ii. How large is "large enough"? → The answer depends on the shape of the population distribution from which we take our sample. (In general, with $n \geq 30$, we are safe)

If the population is far from Normal →

If the population is close to Normal →

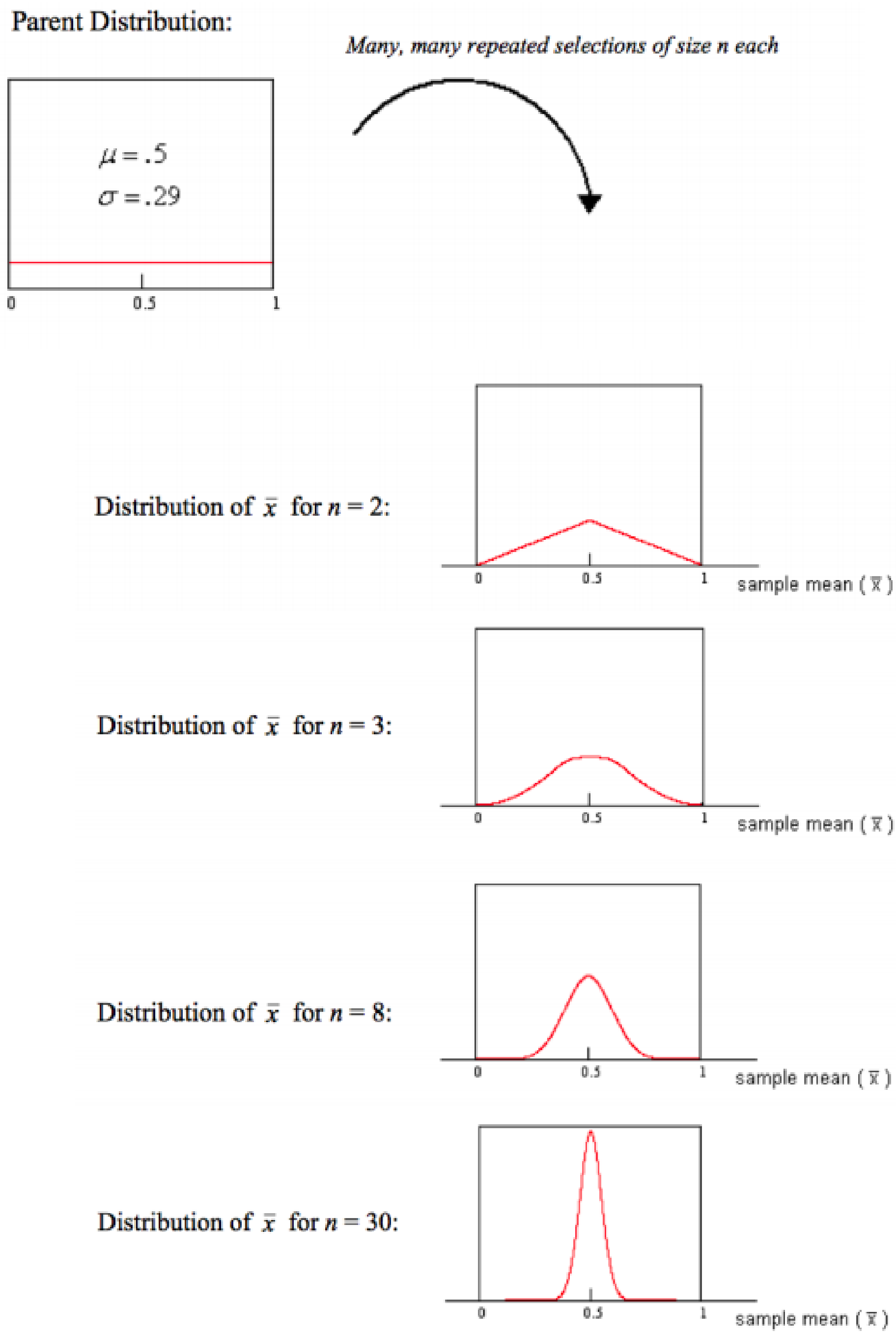
If the population distribution happens to be Normal itself →

iii. Note that averages tend to be ____ ***spread out***, with the spread decreasing by a factor of

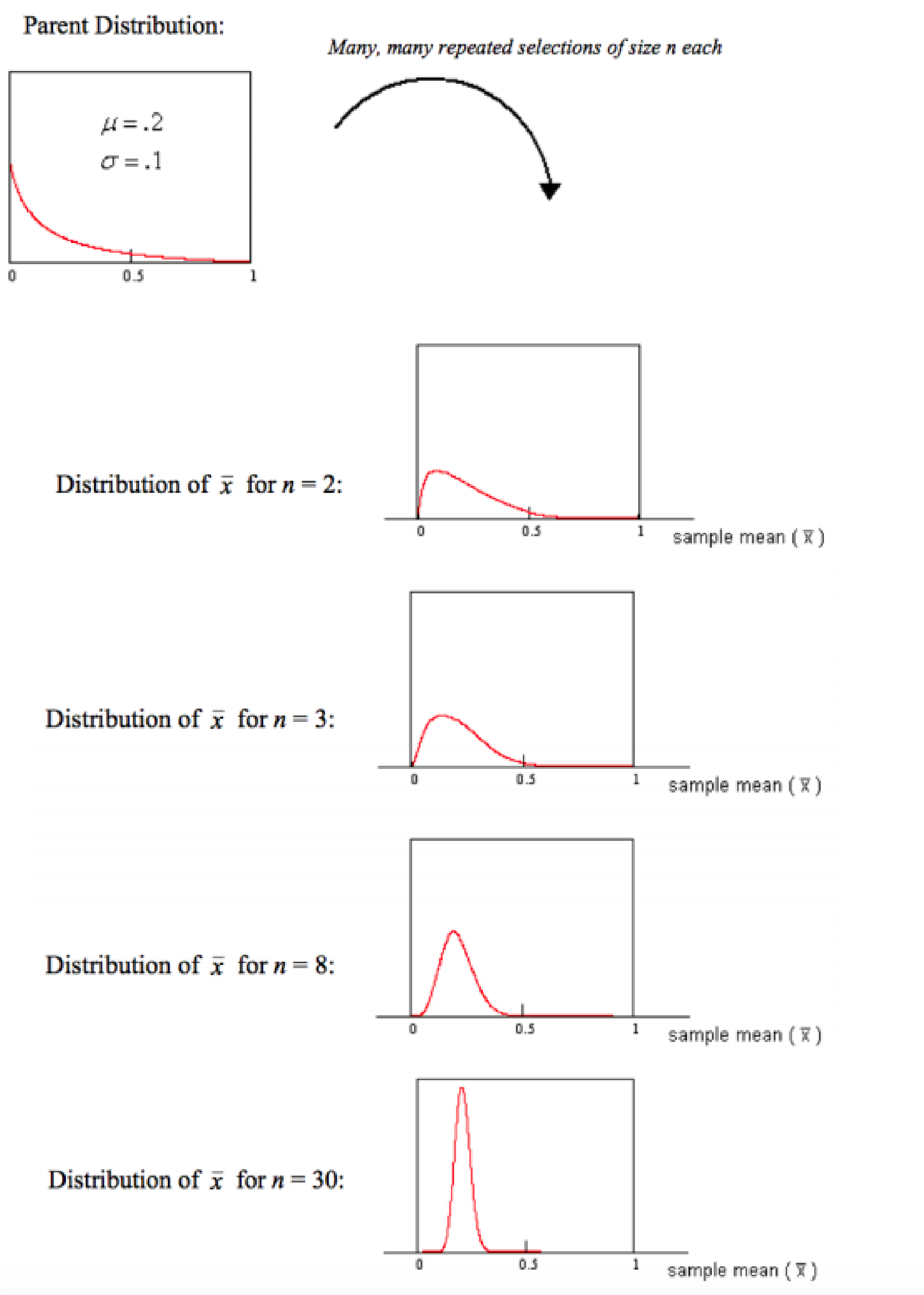
as the sample size n is increasing.

Let's see the Central Limit Theorem in action on 2 specific population distributions:

The Central Limit Theorem in action for the Uniform Distribution



The Central Limit Theorem in action for the Exponential Distribution



Example – Height & Popularity

The heights of male seniors in high school follow a normal distribution with a mean of 74 inches, give or take 2.7 inches. To assess whether there is a connection between height and popularity in high school, the average height ($\bar{x}$) *of the top 5 nominees for prom king at a high school was calculated.*

a) Draw what's going on in the "big picture".

b) What is the sampling distribution of $\bar{x}$? Remember to mention the shape of the distribution, it's mean, and the standard deviation.

c) Draw the sampling distribution of $\bar{x}$ for this example.

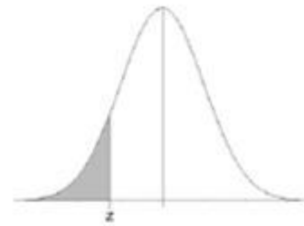
d) Suppose that, in fact, the average height of the 5 students was found to be $\bar{x} = 71.8$. While this is ***some*** evidence that the more popular students tend to be taller, would you say that this is ***enough*** evidence to draw the conclusion that there is a connection between height and popularity in high school? Or is $\bar{x} = 71.8$ just a result that could have happened by chance due to sampling variability?

e) Support your answer to part (d) by calculating the probability of obtaining $\bar{x} = 71.8$ or lower just by chance (i.e. $P(\bar{X} \leq 71.8)$).

Summary of Sampling Distributions

	Sampling distribution of $\hat{p}$	**Sampling distribution of $\bar{\mathbf{X}}$**
Shape:	**Approximately normal** ***Justification:*** $np \geq 10$ $n(1-p) \geq 10$	**Approximately normal** ***Justification:*** $n \geq 30$ **Normal (for any sample size)** ***if population is normal***
Mean:	$\mu_{\hat{p}} = \mathbf{p}$	$\mu_{\bar{x}} = \mu$
Standard deviation:	$\sigma_{\hat{p}} = \sqrt{\dfrac{\mathbf{p(1-p)}}{\mathbf{n}}}$	$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{\mathbf{n}}}$

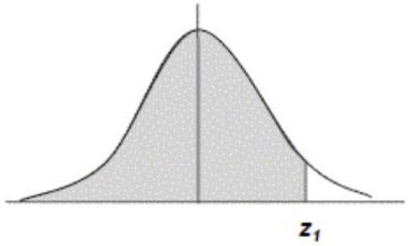
Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Standard Normal Distribution



$$p(z \le z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{1}{2}z^2}\, dz$$

z_1	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000