Lecture 7 – Agenda & Examples

Agenda

1. Review Questions
2. Binomial Distribution as an Extension of Bernoulli Trials
3. Binomial Criteria – "FITS"
4. Expectation & Variance
5. Examples

Review

1. Find the probability distribution for the following
   a. When 2 dice are rolled, the random variable X = # 4's rolled
   b. A fair coin is tossed 2 times. Let the random variable H represent the number of Heads you get. Find the probability distribution of H.
   c. A fair coin is tossed until you get one Head and one Tail, or until the coin has been tossed three times. Let the random variable X represent the number of Tails you got. Find the probability distribution of X. Let the RV Y be the number of tosses. Find the probability distribution of Y.
2. Use the given probability histogram to answer the following questions:
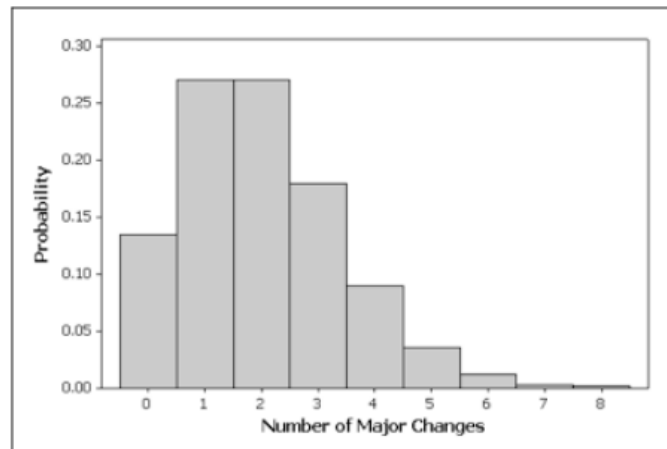
   a. From the probability histogram above, what is the probability that a randomly selected student makes 1 major change?
   b. From the probability histogram above, what is the probability that a randomly selected student makes at least 2 major changes?
3. Consider the given probability distribution for the random variable X measuring the number of hits in a baseball game:

$$P(x) = \frac{x + 2}{20} \quad x = 0,1,2,3,4$$

a. What is the probability that a randomly selected player gets 3 hits?
b. What is the probability that a randomly selected player gets at least 1 hit?
c. Translate the probability distribution from formula form to table form.
d. What is the expected number of hits? What is the variance?
e. If we change the possible values of x to x = 2,4,6,8 is P(x) still a probability distribution?

Lecture

1. Identify whether the following follow a binomial distribution:
    a. A coin is flipped 10 times with the RV Z counting the number of times a tail comes up.
    b. An urn has 20 marbles (10 Red, 7 Blue, 3 Green). 6 marbles are chosen without replacement, let the RV X be the number of Blue marbles pulled.
    c. 5 couples had children until they had 1 boy and 1 girl, or until they had at most 5 children. Let the RV X be the number of boys they had.
    d. A student rolls a weighted dice 10 times (40% roll 1, 20% roll 2, 10% roll 3, 5% roll 4, 5% roll 5, and 20% roll 6). Let the RV Y be the number of times a 1 came up.
2. Suppose that we conduct a random experiment by tossing the same coin 20 times. Let the random variable X be the number of heads tossed in the experiment.
    a) Is X a discrete or continuous random variable?
    b) What are the possible values of X? Is this a subset of the sample space?
    c) Does X follow a binomial distribution, i.e. is X a Binomial Random Variable? If so, properly label the distribution.
    d) What is the probability distribution function of X?
    e) What is the probability that exactly 8 tosses are heads?
    f) What is the probability that less than 3 tosses are heads?
    g) What is the probability that at most 2 tosses are heads?
    h) What is the probability that at least 3 tosses are heads?
    i) Find the expected number or heads to come up.
    j) How much are we "off" by, on average, when we use the mean of X to estimate the number of heads that will come up?
3. Consider the scenario where we roll a die 14 times. Let the RV C be the number of times a "4" comes up.
    a. Does C follow a Binomial distribution? Label the distribution properly and write down the probability distribution function.
    b. Find the $E[X^2]$ in this case.
4. Consider the RV $Y \sim Bin(n, p)$. Recursively define the $p(y)$ by first finding $\frac{p(y)}{p(y-1)}$.