

## Lecture 6 – Agenda & Examples

### Agenda

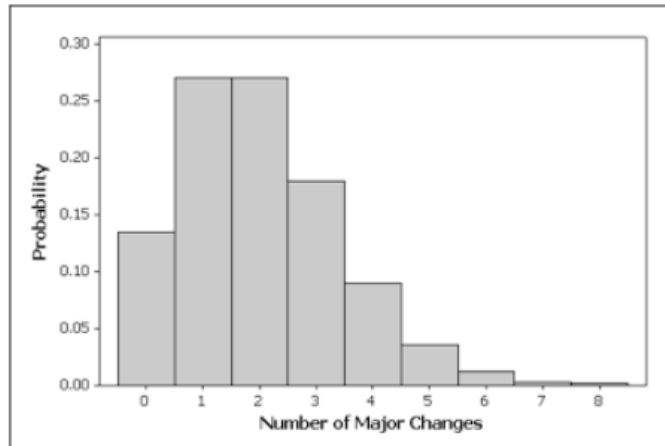
1. Review Questions
2. Random Variables – numerical representation of an outcome of a random experiment
3. Discrete vs Continuous Random Variables (the support of the RV & how to calculate probabilities)
4. Discrete Random Variables – pdf, 2 key properties of all pdfs
5. Forms of Discrete Random Variables – Table (Info Given), Probability Histogram, Formula
6. Expectation, Properties of Expectation, & Variance
7. Examples

### Review

1. The statistics department is reviewing applications for Fall 2019 Graduate Assistants. It typically gets applications primarily from China, India, and the U.S. Over the last few years, the department typically gets 60% from China and 25% from India. Applications are either submitted electronically or mailed in. For those from China, typically 75% are submitted electronically, whereas 60% of those applications from the U.S. are mailed in, and 50% of applications from India are submitted electronically. Suppose a recent application is randomly selected.
  - a. Identify the information given (use events C,I,U,E, and M)
  - b. Draw a probability tree for this information.
  - c. What is the probability that the selected application was submitted by mail?
  - d. What is the probability that the selected application was submitted electronically?
  - e. If the selected application was submitted electronically, what are the probabilities of it being from China, India, or the U.S.?
2. If the events  $A_1, A_2, \dots, A_n$  are mutually exclusive and all equally likely with probability  $p$ , find  $P((A_1 \cap A_2) \cap (\cup_{i=3}^n A_i))$ .
3. Show that if  $A_1, A_2, \dots, A_n$  are equally likely independent events with probability  $p$ , then find  $P(\cap_{i=1}^n A_i | A_2 \cup A_3)$ .

### Lecture

1. Find the probability distribution for the following
  - a. When 2 dice are rolled, the random variable  $X = \# 4$ 's rolled
  - b. A fair coin is tossed 2 times. Let the random variable  $H$  represent the number of Heads you get. Find the probability distribution of  $H$ .
  - c. A fair coin is tossed until you get one Head and one Tail, or until the coin has been tossed three times. Let the random variable  $X$  represent the number of Tails you got. Find the probability distribution of  $X$ . Let the RV  $Y$  be the number of tosses. Find the probability distribution of  $Y$ .
2. Use the given probability histogram to answer the following questions:



- a. From the probability histogram above, what is the probability that a randomly selected student makes 1 major change?
- b. From the probability histogram above, what is the probability that a randomly selected student makes at least 2 major changes?
3. Consider the given probability distribution for the random variable X measuring the number of hits in a baseball game:

$$P(x) = \frac{x + 2}{20} \quad x = 0, 1, 2, 3, 4$$

- a. What is the probability that a randomly selected player gets 3 hits?
- b. What is the probability that a randomly selected player gets at least 1 hit?
- c. Translate the probability distribution from formula form to table form.
- d. What is the expected number of hits? What is the variance?
- e. If we change the possible values of x to x = 2, 4, 6, 8 is P(x) still a probability distribution?
4. Suppose that you work for an insurance company, and you sell a \$150,000 whole-life insurance policy at an annual premium of \$1250. Actuarial tables show that the probability of death during the next year for a person of your customer's age, sex, health, etc. is 0.005.
  - a. Let the RV X be the company's gain (i.e. amount of money made by the company) for a policy of this type. What is the probability distribution function of X in table form?
  - b. What is the expectation with regard to company profit (gain) for a policy of this type?
  - c. Suppose that a decade has passed and your actuarial tables indicate that the probability of death during the next year for a person of your customer's **current** age is .025. This change in probability will obviously be reflected in the annual premium paid. What should the annual premium be (instead of \$1250) if the company intends to keep the same expected profit (gain)?