Lecture 20 – Agenda & Examples

Agenda

1. SET Surveys
2. Review Question
3. Method of Distribution Function
   a. Steps
      i. Find $F_U(u) = P(U < u) = P(h(X) < u)$
      ii. Differentiate $F_U(u)$ to get the pdf of U.
   b. Limitation: Only works for the continuous case
   c. Advantage: The function need not be 1-1.
4. Method of Transformation
   a. Steps
      i. Confirm the function is either increasing or decreasing over the given support
      ii. Find $h^{-1}(u)$
      iii. Find the $f_X\left(h^{-1}(u)\right)$
      iv. Find $\left|\frac{\partial}{\partial u} h^{-1}(u)\right|$
      v. $f_U(u) = f_X\left(h^{-1}(u)\right)\left|\frac{\partial}{\partial u} h^{-1}(u)\right|$
   b. Limitation: Only for monotone functions.
   c. Advantage: Computationally more straight forward.
5. Method of MGF
   a. Steps
      i. Write the MGF of U as a function of the MGF of the known distribution $M_U(t) = E[e^{tU}] = E\left[e^{h(X)t}\right]$
      ii. Identify the form of $M_U(t)$ as that of a known MGF
      iii. Connect this to identify the pdf of U
   b. Limitation: You may not always get a known version of the MGF for $M_U(t)$
   c. Advantage: Works best with independent & identically distributed RV's.
6. Note: For each pset question, please use the method detailed in the section that the question comes from.
7. Examples

Review

1. Suppose that (U_1, U_2) is distributed as $N_2(5, 15, 8, 8, \rho)$ for some $\rho \in (-1,1)$. Let $X_1 = U_1 + U_2$ and $X_2 = U_1 - U_2$. Show that X_1 and X_2 are uncorrelated.

Lecture

1. Consider the following pdf:

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & o.w. \end{cases}$$

a. Let $U = 2X - 1$. Find the pdf of U.
b. Let $V = X^2$. Find the pdf of V. How does this change if $V = X^3$?

2. (Text, Exercise 6.8) Assume that $Y \sim Beta(\alpha, \beta)$.
   a. Find the density function, $f_U(u)$, of $U = 1 - Y$ using the density function approach. Properly label the distribution $f_U(u)$
   b. Find the density using the transformation approach (first verify the function is monotone). Properly label the distribution.
   c. How is $E[U]$ related to $E[Y]$?
   d. How is $V[U]$ related to $V[Y]$?
3. Consider the following joint density function

$$f(x, y) = \begin{cases} 2(1-x), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & o.w. \end{cases}$$

   Find the pdf for $U = XY$. (Note: this is still univariate since we are only transforming to U. If we also had $V = \frac{Y}{X}$, for example and were interested in the joint pdf $f_{U,V}(u, v)$ then this would be a multivariate transformation and requires the Jacobian matrix (next lecture)).
4. (Textbook Exercise 6.43) Let $Y_1, Y_2, \dots, Y_n$ be i.i.d. (independent and identically distributed) normal random variables with common distribution $Y_i \sim N(\mu, \sigma^2)$.
   a. Find the density function of $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ (think about the sampling distribution of sample means from your introductory course…look familiar?)
   b. If $\sigma^2 = 16$ and $n = 25$, what is the probability that the sample mean $\bar{Y}$ takes on a value that is within one unit of the population mean μ? (i.e., find $P(|\bar{Y} - \mu| \le 1)$).