Lecture 10 – Agenda & Examples

Agenda

1. Review Questions
2. Moments → $E[X^k]$
3. Moment Generating Function → $M_X(t) = E[e^{tX}]$
4. How to use the MGF → $E[X^k] = \left[\frac{d^k}{dt^k} M_X(t)\right]_{t=0}$
5. Bernoulli, Binomial, Geometric, Poisson
6. Uniqueness of the MGF
7. Examples

Review

1. Calculate the variance of the RV X if it follows a Poisson distribution with a mean of 3.
2. Suppose that a random variable X has the $Poisson(\lambda)$ distribution, $0 < \lambda < \infty$. For all $x = 0,1,2,\ldots$, show that $\frac{P(X=x)}{P(X=x+1)} = \frac{(x+1)}{\lambda}$.
3. A jury of 6 people was selected from a group of 20 potential jurors one at a time, of whom 8 were African American and 12 were white. The jury was supposedly randomly selected, but it contained only 1 African American member. Do you have any reason to doubt the randomness of the selection?
4. Derive the expectation of the RV $X \sim Hypergeo(N, r, n)$ to be $E[X] = r\frac{\binom{N-1}{n-1}}{\binom{N}{n}} = \frac{nr}{N}$

Lecture

1. Derive the variance of the Binomial distribution.
2. Derive the mean of the geometric distribution.
3. Derive the variance of the Poisson distribution.
4. If $X \sim Bin(n, p)$, find the mean and variance of $Y = 3X$.