Lecture 9 – Agenda & Examples

Agenda

0. Proof Strategy & Mindset
1. Review Questions
2. Hypergeometric Distribution – Context & Proof of Valid pdf $\left(X \sim Hypergeo(N, r, n) \rightarrow p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}\right)$
3. Poisson Distribution – Context & Proof of Valid pdf $\left(X \sim Poisson(\lambda) \rightarrow p(x) = \frac{\lambda^x e^{-\lambda}}{x!}\right)$
4. Expectation & Variance – Proof of Poisson Mean $(X \sim Poisson(\lambda) \rightarrow E[X] = V[X] = \lambda)$ $\left(X \sim Hypergeo(N, r, n) \rightarrow E[X] = \frac{nr}{N} \text{ and } V[X] = n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)\right)$
5. Recap of Discrete Random Variables
6. Examples

Review

1. Recursively define the $p(x)$ if $X \sim Nbin(r, p)$.
2. If Y has a geometric distribution with success probability p, show that $P(Y = odd\ integer) = \frac{p}{1-q^2}$ where $q = 1 - p$.
3. Let Y denote a geometric random variable with probability of success p. Show that for positive integers a and b, $P(Y > a + b | Y > a) = q^b = P(Y > b)$.

Lecture

1. Calculate the variance of the RV X if it follows a Poisson distribution with a mean of 3.
2. Suppose that a random variable X has the $Poisson(\lambda)$ distribution, $0 < \lambda < \infty$. For all $x = 0,1,2, \ldots$, show that $\frac{P(X=x)}{P(X=x+1)} = \frac{(x+1)}{\lambda}$.
3. A jury of 6 people was selected from a group of 20 potential jurors one at a time, of whom 8 were African American and 12 were white. The jury was supposedly randomly selected, but it contained only 1 African American member. Do you have any reason to doubt the randomness of the selection?
4. Derive the expectation of the RV $X \sim Hypergeo(N, r, n)$ to be $E[X] = r\frac{\binom{N-1}{n-1}}{\binom{N}{n}} = \frac{nr}{N}$.