Counting Techniques Practice Example Solutions

Question 2 pt (d)

The key to this question is finding $\frac{\#\ ways\ the\ event\ of\ interest\ can\ happen}{total\ number\ of\ ways\ possible}$. Here, it is most helpful to think about which cases satisfy the scenario of drawing a King of hearts and queen of hearts simultaneously. This can only happen if we select KQ or QK. So, the numerator is 2. The total number of ways that we can select 2 cards simultaneously is $\binom{52}{2}$. So, the answer is $\frac{\#\ ways\ the\ event\ of\ interest\ can\ happen}{total\ number\ of\ ways\ possible} = \frac{2}{\binom{52}{2}} = \frac{2}{(52)(51)} = \frac{1}{1326}$

Question 2 pt (e)

Again, the key to this question is finding $\frac{\#\ ways\ the\ event\ of\ interest\ can\ happen}{total\ number\ of\ ways\ possible}$. Let's think about which cases satisfy the scenario of the 2 cards being dealt simultaneously sum to 1 (assume Ace is 1). This can happen 22 ways. We can have the following:

A_H3_H	A_S3_H	A_D3_H	A_C3_H	2_C2_H	2_D2_S
A_H3_S	A_S3_S	A_D3_S	A_C3_S	2_C2_S	2_S2_H
A_H3_D	A_S3_D	A_D3_D	A_C3_D	2_C2_D	
A_H3_C	A_S3_C	A_D3_C	A_C3_C	2_D2_H	

So, the numerator is 22. The total number of ways that we can select 2 cards simultaneously is $\binom{52}{2}$. So, the answer is $\frac{\#\ ways\ the\ event\ of\ interest\ can\ happen}{total\ number\ of\ ways\ possible} = \frac{22}{\binom{52}{2}} = \frac{22}{(52)(51)} = \frac{22}{1326} = \frac{11}{663}$

Question 3 pt (a)

As always, we are finding $\frac{\#\ ways\ the\ event\ of\ interest\ can\ happen}{total\ number\ of\ ways\ possible}$. For this question, the numerator is the number of ways we can choose 2 yellow balls from all 3 yellow balls, i.e. $\binom{3}{2} = 3$. The denominator is the total number of ways we can choose 2 balls from the 9 total, i.e. $\binom{9}{2} = 36$.

So, the answer is $\frac{\#\ ways\ the\ event\ of\ interest\ can\ happen}{total\ number\ of\ ways\ possible} = \frac{3}{36} = \frac{1}{12}$

Question 3 pt (b)

As always, we are finding $\frac{\#\ ways\ the\ event\ of\ interest\ can\ happen}{total\ number\ of\ ways\ possible}$. For this question, the numerator is the number of ways we can choose 1 red and 1 green ball, i.e. $\binom{3}{1}\binom{3}{1} = 9$. The denominator is the total number of ways we can choose 2 balls from the 9 total, i.e. $\binom{9}{2} = 36$.

So, the answer is $\frac{\#\ ways\ the\ event\ of\ interest\ can\ happen}{total\ number\ of\ ways\ possible} = \frac{9}{36} = \frac{1}{4}$

Question 3 pt (c)

As always, we are finding $\frac{\#\ ways\ the\ event\ of\ interest\ can\ happen}{total\ number\ of\ ways\ possible}$. For this question, the numerator is the number of ways we can choose marbles of different colors. This can happen if we have RY, RG, or YG. So the numerator is RY+RG+YG =$\binom{3}{1}\binom{3}{1}+\binom{3}{1}\binom{3}{1}+\binom{3}{1}\binom{3}{1}=9+9+9=27$. The denominator is the total number of ways we can choose 2 balls from the 9 total, i.e. $\binom{9}{2}=36$.

So, the answer is $\frac{\#\ ways\ the\ event\ of\ interest\ can\ happen}{total\ number\ of\ ways\ possible}=\frac{27}{36}=\frac{3}{4}$

Question 3 pt (d)

Now, we have 4 Red, 2 Yellow, and 3 Green. As always, we are finding $\frac{\#\ ways\ the\ event\ of\ interest\ can\ happen}{total\ number\ of\ ways\ possible}$. For this question, the numerator is the number of ways we can choose marbles of different colors. This can happen if we have RY, RG, or YG. So the numerator is RY+RG+YG =$\binom{4}{1}\binom{2}{1}+\binom{4}{1}\binom{3}{1}+\binom{2}{1}\binom{3}{1}=8+12+6=26$. The denominator is the total number of ways we can choose 2 balls from the 9 total, i.e. $\binom{9}{2}=36$.

So, the answer is $\frac{\#\ ways\ the\ event\ of\ interest\ can\ happen}{total\ number\ of\ ways\ possible}=\frac{26}{36}=\frac{13}{18}$