

$$\begin{aligned}
& X \sim \text{Hypergeo}(N, r, n) \\
E[X(X-1)(X-2)] &= \sum_{x=0}^n \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \sum_{x=0}^n X(X-1)(X-2) \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} \\
&= \sum_{x=0}^n X(X-1)(X-2) \frac{\frac{r!}{x!(r-x)!} \binom{N-r}{n-x}}{\binom{N}{n}} \\
&= \sum_{x=3}^n X(X-1)(X-2) \frac{\frac{r!}{x!(r-x)!} \binom{N-r}{n-x}}{\binom{N}{n}} + 0 + 0 + 0 \\
&= \sum_{x=3}^n X(X-1)(X-2) \frac{\frac{r!}{x!(r-x)!} \binom{N-r}{n-x}}{\binom{N}{n}} = \sum_{x=3}^n \frac{\frac{r!}{(x-3)!(r-x)!} \binom{N-r}{n-x}}{\binom{N}{n}} \\
&= r(r-1)(r-2) \sum_{x=3}^n \frac{\frac{(r-3)!}{(x-3)!((r-3)-(x-3))!} \binom{N-r}{n-x}}{\binom{N}{n}} \\
&= r(r-1)(r-2) \sum_{x=3}^n \frac{\binom{r-3}{x-3} \binom{N-r}{n-x}}{\binom{N}{n}} = (\text{with } y = x-3) \\
&= r(r-1)(r-2) \sum_{y=0}^{n-3} \frac{\binom{r-3}{y} \binom{N-r}{n-3-y}}{\binom{N}{n}} = r(r-1)(r-2) \frac{\binom{N-3}{n-3}}{\binom{N}{n}} \\
&= r(r-1)(r-2) \frac{(N-3)!}{\frac{(n-3)!(N-3-n+3)!}{N!}} \\
&= r(r-1)(r-2) \frac{\frac{n!(N-n)!}{n(n-1)(n-2)}}{N(N-1)(N-2)}
\end{aligned}$$