Lecture 11 – Midterm Review

1. If $A_1, A_2, \dots, A_n$ are equally likely independent events with probability p, then find $P[\cap_{i=1}^{n} A_i | A_4]$. How does your answer change if instead of being equally likely independent events, they are mutually disjoint?
2. What is the probability of getting a 5-card straight in poker? (Ace is low).
3. Consider the case where we have an urn of 20 marbles (6 green, 3 purple, and 11 pink). What is the probability when selecting 4 marbles at random (at once), that all 4 are the same color? What about the probability of 3 being the same color?
4. Assume pens can come in only 3 colors: Blue, red, and green. The probability of a pen writing in blue is .60, and in red is .30. When the pen writes in blue, the probability that I will like it is .75. When the pen writes in red, the probability that I won't like it is .60. When it writes in green, the probability that I will like it is .50.
    a. Identify the given info using events B, R, G, and L.
    b. Draw a probability tree for this info
    c. What is the probability that I won't like a randomly selected pen?
    d. If I like the pen, what is the probability that it writes in red?
5. Confirm visually whether $P(A) = P(A \cap B) + P(A^c \cap B)$.
6. Define the context in which we use each type of discrete RV that we discussed.
7. If $X \sim Hypergeo(N, r, n)$, find the $E[X(X-1)(X-2)]$.
8. Prove the geometric distribution has a valid probability distribution function.
9. Consider the following probability distribution functions for the RV's X and Y:

X	1	3	5	7
Y	0	1	2	3
P(X)	.32	.38	.11	.19

    a. Does the RV Y have a valid pdf?
    b. Find $E[Y^2 - 4]$.
    c. What do you think $E[XY]$ is, using the same logic as in part (b)?
10. Derive the 3rd moment of the Bernoulli distribution.
11. Prove that the Poisson distribution is valid.
12. Prove the variance of the Poisson distribution.
13. Derive the MGF of the geometric distribution. What is the $E[X]$ and the bounds on t?
14. Find the distribution of the RV Y with the MGF $m_Y(t) = \frac{e^t}{2 - e^t}$
15. If W = 3Y, show that the MGF of W is $m_Y(3t)$. Then, use the MGF of W to show that $E[W] = 3E[Y]$.