Lecture 19 – Agenda & Examples

Agenda

1. Review Questions
2. If X_1 and X_2 are independent, then $Var[X_1 + X_2] = Var[X_1 - X_2] = Var[X_1] + V[X_2]$
3. Covariance – numerically summarizing how closely related 2 RV's are ($Cov(X_1, X_2) = E[(X_1 - E[X_1])(X_2 - E[X_2])]$
4. Alternative formula for Covariance ($Cov(X_1, X_2) = E[X_1X_2] - E[X_1]E[X_2]$)
5. If two random variables are independent, $Cov(X_1, X_2) = 0$
6. Correlation $\left(\rho = Corr(X_1, X_2) = \frac{Cov(X_1,X_2)}{\sqrt{Var[X_1]}\sqrt{Var[X_2]}}\right)$
7. Linear Combinations ($U_X = \sum_{i=1}^{n} a_i X_i$ and $U_Y = \sum_{j=1}^{m} a_j Y_j$
   a. Expectation → $E[U_X] = \sum_{i=1}^{n} a_i E[X_i]$
   b. Variance → $Var[U_X] = \sum_{i=1}^{n} a_i^2 Var[X_i] + \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j Cov\left[X_i, X_j\right]$ for i≠j
   c. Covariance → $Cov[U_X, U_Y] = \sum_{i=1}^{n}\sum_{j=1}^{m} a_i b_j Cov\left[X_i, Y_j\right]$
8. Examples

Review

1. Consider the joint density below:
$$f(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)}, & x_1 > 0, x_2 > 0 \\ 0, & o.w. \end{cases}$$
   a. Are X_1 and X_2 independent?
   b. What is the $E[X_1^2 X_2^3]$?
   c. What is the $E[(X_1 - 4)^2(X_2 + 1)]$?
   d. What is the $V[X_1 + X_2]$ using the variance shortcut formula? How about using the properties of independence? What do you notice?
   e. What is the $P(X_1 - X_2 < -3)$?
2. Suppose that X_1 and X_2 are independent χ^2 random variables with degrees of freedom v_1 and v_2, respectively.
   a. Find $E[X_1 + X_2]$
   b. Find $V[X_1 + X_2]$
3. Consider the following joint pdf
$$f(x_1, x_2) = \begin{cases} \frac{1}{8} x_1 e^{-\frac{(x_1+x_2)}{2}}, & 0 < x_1, x_2 \\ 0, & o.w. \end{cases}$$
   a. Find the $E[x_1 x_2]$
   b. Find $E[x_1^2 - 2x_1x_2 + x_2^2]$
   c. What is the $Var[x_1 + x_2]$?
4. Consider the following joint pdf

X_2 Values	**X_1 Values**	
	0	**1**

0	.38	.17
1	.14	.02
2	.24	.05

a. Are X_1 and X_2 independent?
b. Find the $Var(X_1^2 - X_2^3)$

Lecture

1. Prove the following:
   a. $Cov[X_1, c] = 0$ for any constant c
   b. $Cov[X_1, X_1] = Var[X_1]$
   c. $Cov[X_1, X_2] = Cov[X_2, X_1]$
   d. $Cov[X_1 + X_2, Y_1 + Y_2] = Cov[X_1, Y_1] + Cov[X_1, Y_2] + Cov[X_2, Y_1] + Cov[X_2, Y_2]$ (this proves that covariance is a bilinear operation, implying that it is a linear operation in both coordinates)
2. Prove that if $Y_i = c_i + d_i X_i$, then $\rho_{Y_1,Y_2} = \rho_{X_1,X_2}$ for constants c_i, d_i
3. Let the discrete RV's Y_1 and Y_2 have the joint probability function $p(y_1, y_2) = \frac{1}{3}$ for $(y_1, y_2) = (-1,0), (0,1), (1,0)$.
   a. Find the $Cov[Y_1, Y_2]$ and ρ_{Y_1,Y_2}.
   b. Are Y_1 and Y_2 independent? (what do you notice about what correlation implies about independence? It only goes 1 way!)
4. (Text 5.113) A retail grocery merchant figures that her daily gain X from sales is a normally distributed RV with mean 50 and standard deviation 3 (measurement in dollars). X can be negative if she is forced to dispose of enough perishable goods. Also, she figures daily overhead costs Y to have a gamma distribution with alpha of 4 and beta of 2. If X and Y are independent, find the expected value and variance of her net daily gain. Would you expect her net gain for tomorrow to rise above $70?