

Lecture 14 – Agenda & Examples

Agenda

1. Review Questions
2. Normal Distribution $\left(f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad x \in \mathbb{R} \right)$
 - a. Proof of valid pdf
 - b. Proof of expectation (variance and MGF left as homework)
3. Standardization
4. Empirical Rule
5. Binomial Approximation – Conditions ($np \geq 10$ and $n(1 - p) \geq 10$) & Distribution $(X \sim N(np, np(1 - p)))$
6. Continuity Correction – using a continuous RV to approximate a discrete random variable $(P(X \geq a) = P(X \geq a - 0.5))$
 - a. The idea: Maximize the area that you are considering under the normal distribution such that the inequality still represents the same possible values of X under the binomial distribution.
 - i. $P(X \leq 120) = P(X \leq 120.5)$
 - ii. $P(X < 120) = P(X \leq 119) = P(X \leq 119.5)$
 - iii. $P(130 \leq X \leq 145) = P(129.5 \leq X \leq 145.5)$
7. Examples

Review

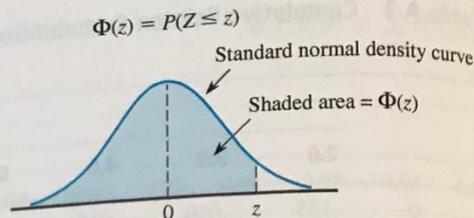
1. Suppose that the random variable $Y \sim \chi_v^2$
 - a. Give an expression for $E(Y^a)$ if $v > -2a$
 - b. Why did the answer in part (a) require $v > -2a$?
 - c. Give an expression for $E(\sqrt{Y})$. What do you need to assume about v ?
 - d. Give an expression for $E(\sqrt[5]{Y^3})$. What do you need to assume about v ?
2. Derive the MGF of the exponential distribution. How must we restrict t in order to obtain realistic moments?
3. The length of time X necessary to complete a key operation in the construction of houses has an exponential distribution with mean 10 hours. The formula $C = 100 + 40X + 3X^2$ relates the cost C of completing this operation to the square of the time to completion. Find the mean and variance of C.
4. Find the median of the random variable X if X follows an exponential distribution with 4th moment of $\frac{1}{54}$.

Lecture

1. Show that the maximum value of the normal density with parameters μ and σ is $\frac{1}{\sigma\sqrt{2\pi}}$ and occurs when $x = \mu$.

2. Let Y be normally distributed with mean 4 and variance 1. Find the following:
 - a. The range of values seen as “typical”
 - b. The 84th percentile
 - c. What percentage of observations fall below 2?
 - d. What percentage of points fall above 5?
3. Let X be normally distributed with mean 10 and standard deviation 5. Find the following probabilities:
 - a. $P(X \leq 8)$
 - b. $P(X \geq 11)$
 - c. The 86th percentile
 - d. Q1 of this distribution
 - e. The median of this distribution
 - f. The value such that 16% of the data fall above this point.
 - g. $P(1 \leq |X|)$
4. Suppose that 10% of all steel shafts produced by a certain process are nonconforming but can be reworked (rather than having to be scrapped). Consider a random sample of 200 shafts, and let X denote the number among these that are nonconforming and can be reworked. What is the (approximate) probability that X is
 - a. At most 30?
 - b. Less than 30?
 - c. Between 15 and 25 (inclusive)?

Table A.3 Standard Normal Curve Areas



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued)

