Austin Menger
3375Q

Lecture 11 & 12 – Agenda & Examples

Agenda

1. Probability Density Functions, Probability Distribution Functions, and Probability Mass Functions
2. 2 Distinguishing properties of continuous random variables – support is an interval & $P(X = x) = 0$ for any singular value of x in the support.
3. Finding probabilities using a pdf ($P(a \le X \le b) = \int_a^b f(x)dx$)
4. 2 Properties of every legitimate pdf $f(x) \ge 0 \ \forall x \in \chi$ & $\int_{-\infty}^{\infty} f(x) = 1$
5. CDF $F(x) = P(X \le x) = \int_{-\infty}^{x} f(y)dy$
6. Expectation, Variance, moments, and MGF's
7. Uniform Distribution
   a. Context
   b. pdf derivation($\int_A^B c\, dx$)
   c. cdf derivation (for $X \sim U(a,b)$ $F(x) = \frac{x-a}{b-a}$)
   d. Expectation and Variance Derivation ($E[X] = \frac{a+b}{2}$ $V[X] = \frac{(b-a)^2}{12}$)
   e. Percentiles
   f. MGF Derivation
8. Examples

Lecture

1. Verify whether the following functions are valid probability density functions on the given support
   i. $f(x) = \begin{cases} .075x + .2 & 3 \le x \le 5 \\ 0 & otherwise \end{cases}$
   ii. $f(x) = \begin{cases} x - 4 & 1 \le x \le 4 - \sqrt{11} \\ 0 & otherwise \end{cases}$
   iii. $f(x) = \begin{cases} \frac{1}{25}y & 0 \le y < 5 \\ \frac{2}{5} - \frac{1}{25}y & 5 \le y \le 10 \\ 0 & y < 0 \ or \ y > 10 \end{cases}$
2. Consider the function $f(x) = \begin{cases} kx^3 + \frac{k}{2}x^2 & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$
   a. Find k such that the above function is a valid probability density function on the given support.
   b. What is the probability that $x \in (0,1)$? Is this the same as the probability $x \in [0,1]$? Why is this so?
3. The cycle time for trucks hauling concrete to a highway construction site is uniformly distributed over the interval 50 to 70 minutes.
   a. What is the probability that the cycle time exceeds 65 minutes if it is known that the cycle time exceeds 55 minutes?

   b. Calculate the IQR of this dataset
4. Suppose that Y has a uniform distribution over the interval [0,1].
   a. Prove that the distribution of Y is valid
   b. What is the third moment, i.e. $E(X^3)$?
   c. What is $E[X^3 - X^2 - 1]$?
5. A telephone call arrived at a switchboard at random within a 1-minute interval. The switch board was fully busy for 15 seconds into this 1-minute period. What is the probability that the call arrived when the switchboard was not fully busy?
6. Consider the following pdf

$$f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ \frac{1}{2} & 1 < x \leq 2 \\ 0 & o.w. \end{cases}$$

   a. Is the distribution differentiable everywhere? How about continuous?
   b. Find $F(x)$